24. $f(x) = 1 + \frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} \cdots$

$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^3(0)}{3!}x^3 \cdots$

a) $f'(0) = \frac{1}{2}$

$\frac{f''(0)}{\cancel{2}} = \frac{1}{3!} \cdot 2$

$f^{10}(0) = \frac{10!}{11!} = \frac{1}{11}$

$\frac{f^3(0)}{\cancel{3!}} = \frac{1}{4!} \cdot 3!$

$\frac{10!}{11!} = \frac{\cancel{10 \cdot 9 \cdot 8 \cdot 7 \cdots 1}}{11 \cdot \cancel{10 \cdot 9 \cdot 8 \cdot 7 \cdots 1}}$

b $g(x) = x\, f(x)$

$= x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots = e^x - 1$ (c)

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots$

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25. b $\frac{e^x - 1}{x}$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$ n starts at 0

$g(x) = \frac{e^x - 1}{x} = \frac{x}{x} + \frac{x^2}{x\,2} + \frac{x^3}{x\,3!} + \frac{x^4}{x\,4!} + \cdots + \frac{x^n}{x\,n!}$

n starts at 1

$g(x) = 1 + \frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \frac{x^{n-1}}{n!}$

c) $g'(x) = \frac{1}{2} + \frac{2x}{3!} + \frac{3x^2}{4!} + \cdots$

$g'(1) = \frac{1}{2} + \frac{2}{3!} + \frac{3}{4!} + \cdots = \sum_{n=1}^{\infty} \frac{n}{(n+1)!}$

$g'(x) = \frac{x\,e^x - (e^x - 1) \cdot 1}{x^2}$

$g'(1) = \frac{e - e + 1}{1} = 1$

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9.2b Taylor Series

Maclaurin series works for values of x near 0

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 \ldots$$

$$\boxed{\begin{array}{l}\text{Taylor series works for values of } x \text{ near } a \\ = f(a) + f'(a)(x-a) + \dfrac{f''(a)}{2}(x-a)^2 + \dfrac{f'''(a)}{3!}(x-a)^3 \ldots \\ = \displaystyle\sum_{n=0}^{\infty} \frac{f^{n}(a)(x-a)^n}{n!}\end{array}}$$

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find the 1st 4 terms of the Taylor Series for $f(x) = e^x$ about $x=2$

at $x=2$

centered at $x=2$

$a=2$

$f(x) = e^x \quad f(2) = e^2$

$f'(x) = e^x \quad f'(2) = e^2$

$f''(x) = e^x \quad f''(2) = e^2$

$$e^x \approx e^2 + e^2(x-2) + \frac{e^2}{2}(x-2)^2 + \frac{e^2}{3!}(x-2)^3 \ldots \frac{e^2}{n!}(x-2)^n$$

$f(x) = \cos x$ 1st 3 terms about $x = \frac{\pi}{4}$

$f'(x) = -\sin x \quad f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$f''(x) = -\cos x \quad f'\left(\pi/4\right) = -\sqrt{2}/2$

$f''\left(\pi/4\right) = -\sqrt{2}/2$

$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{2\cdot 2}\left(x-\frac{\pi}{4}\right)^2$$

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find the Maclaurin series for $\frac{1+\cos(2x)}{2}$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots$$

$$\cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} \dots$$

$$\frac{1+\cos(2x)}{2} = \frac{2}{2} - \frac{(2x)^2}{2\cdot 2!} + \frac{(2x)^4}{2\cdot 4!} - \frac{(2x)^6}{2\cdot 6!} + \dots \frac{(-1)^n (2x)^{2n}}{2\cdot(2n)!}$$

start at $n=0$

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